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3. The continuous random variable X is uniformly distributed over the interval $[-1,3]$. Find

- (a) $E(X)$ (1)

- (b) $\text{Var}(X)$ (2)

- (c) $E(X^2)$ (2)

- (d) $P(X < 1.4)$ (1)

A total of 40 observations of X are made.

(e) Find the probability that at least 10 of these observations are negative. (5)



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5. A continuous random variable X has the probability density function $f(x)$ shown in Figure 1.

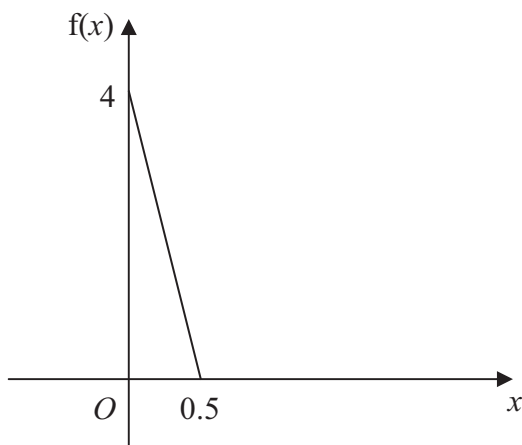


Figure 1

- (a) Show that $f(x) = 4 - 8x$ for $0 \leq x \leq 0.5$ and specify $f(x)$ for all real values of x . **(4)**
- (b) Find the cumulative distribution function $F(x)$. **(4)**
- (c) Find the median of X . **(3)**
- (d) Write down the mode of X . **(1)**
- (e) State, with a reason, the skewness of X . **(1)**



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6. Cars arrive at a motorway toll booth at an average rate of 150 per hour.
- (a) Suggest a suitable distribution to model the number of cars arriving at the toll booth, X , per minute. (2)
 - (b) State clearly any assumptions you have made by suggesting this model. (2)
- Using your model,
- (c) find the probability that in any given minute
 - (i) no cars arrive,
 - (ii) more than 3 cars arrive. (3)
 - (d) In any given 4 minute period, find m such that $P(X > m) = 0.0487$ (3)
 - (e) Using a suitable approximation find the probability that fewer than 15 cars arrive in any given 10 minute period. (6)



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Question 6 continued

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7. The queuing time in minutes, X , of a customer at a post office is modelled by the probability density function

$$f(x) = \begin{cases} kx(81-x^2) & 0 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{4}{6561}$. **(3)**

Using integration, find

(b) the mean queuing time of a customer, **(4)**

(c) the probability that a customer will queue for more than 5 minutes. **(3)**

Three independent customers shop at the post office.

(d) Find the probability that at least 2 of the customers queue for more than 5 minutes. **(3)**



